Exam #1: Chapter 8 NAME Answer Key
Math 178, Section 0663
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100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

1. (4 pts each) Compute the following limits **without using graphs or tables**:

A. $\lim_{x\to 1}\frac{x-1}{3x^2-x-2}$

$\lim_{x\to 1}\frac{\cancel{x-1}}{\cancel{(x-1)}(3x+2)} = \frac{1}{3x+2} = \boxed{\frac{1}{5}}$

B. $\lim_{x\to 4}\left(\frac{8-x^{1/2}}{x^2}\right) = \lim_{x\to 4}\frac{8-\sqrt{4}}{4^2} = \frac{8-2}{16} = \frac{6}{16} = \boxed{\frac{3}{8}}$

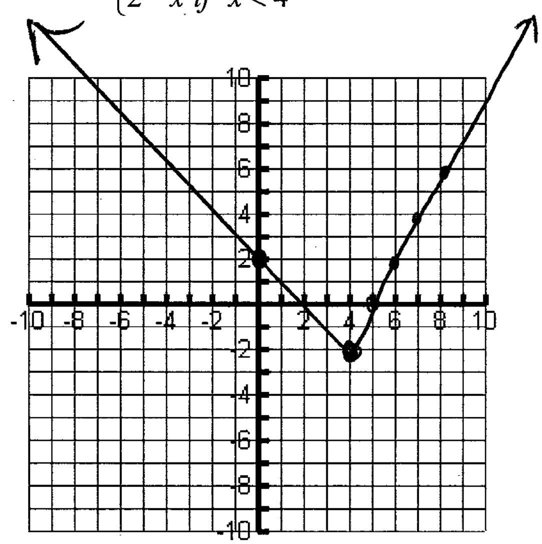
C. $\lim_{h\to 0}\frac{10x^3h^2-5xh}{h}$

$\lim_{h\to 0} 10x^3h - 5x = \boxed{-5x}$

2. (14 pts) Consider the following piecewise function:

Graph $f(x)$ by hand and find the following:

$f(x)=\begin{cases}2x-10 \text{ if } x\ge 4\\ 2-x \text{ if } x<4\end{cases}$



A. $\lim_{x\to 4^-} f(x) = -2$

B. $\lim_{x\to 4^+} f(x) = -2$

C. $\lim_{x\to 4} f(x) = -2$

D. $f(4) = -2$

E. Is $f(x)$ continuous? If it is not, then state the first condition from the definition of continuity that is violated.

YES IT IS CONTINUOUS

3. (10 pts) Use the definition of the derivative, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, to find $f'(x)$ given $f(x)=-2x^2+3x-1$. **(SHOW EVERY STEP LEADING YOU TO YOUR ANSWER TO RECEIVE CREDIT.)**

$$f'(x)=\lim_{h\to 0}\frac{-2(x+h)^2+3(x+h)-1-(-2x^2+3x-1)}{h}$$

$$=\lim_{h\to 0}\frac{-2(x^2+2xh+h^2)+3x+3h-1+2x^2-3x+1}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{-2x^2}-4xh-2h^2+\cancel{3x}+3h\cancel{-1}+\cancel{2x^2}\cancel{-3x}\cancel{+1}}{h}$$

$$=\lim_{h\to 0}\frac{-4xh-2h^2+3h}{h}=\lim_{h\to 0}-4x-2h+3$$

$$=\boxed{-4x+3}$$

4. (10 pts total) A Company's cost function is $C(x)=20+3x+\frac{54}{\sqrt{x}}$ dollars where x is the number of units (for $5\le x\le 20$).

A. (4 pts) Find the Marginal Cost Function.

$$C(x)=20+3x+54x^{-1/2}$$

$$MC(x)=C'(x)=3+54\left(\frac{-1}{2}\right)x^{-3/2}$$

$$MC(x)=\boxed{C'(x)=3-27x^{-3/2}}$$

B. (4 pts) Find the Marginal Cost Function at $x=16$.

$$MC(16)=3-\frac{27}{(\sqrt{16})^3}=3-\frac{27}{64}=2.58$$

C. (2 pts) Interpret your answer from part B.

WHEN 16 UNITS ARE PRODUCED, THE COST IS INCREASING AT THE RATE OF \$2.58 PER UNIT PRODUCED.

5. (30 pts total) Find the derivative using the **Product, Quotient, or Chain rules**, then <u>simplify</u> whenever possible.

A. $f(x)=\dfrac{x^4+1}{x^4-1}$ $\begin{matrix} f \\ g \end{matrix}$ $\qquad \dfrac{f'g-gf'}{g^2}$

$$f'(x) = \frac{4x^3(x^4-1) - 4x^3(x^4+1)}{(x^4-1)^2} = \frac{4x^7-4x^3-4x^7-4x^3}{(x^4-1)^2}$$

$$\boxed{f'(x) = \frac{-8x^3}{(x^4-1)^2}}$$

B. $f(x)=\overset{f}{(x^2+4)}\overset{g}{(x^2-4)}$ $\qquad f'g+g'f$

$$f'(x) = 2x(x^2-4) + 2x(x^2+4)$$
$$= 2x^3-8x + 2x^3+8x$$

$$\boxed{f'(x) = 4x^3}$$

C. $f(x)=\dfrac{1}{\sqrt[5]{(5x+1)^3}}$ $= (5x+1)^{-3/5}$

$$f'(x) = -\frac{3}{5}(5x+1)^{-8/5}(5)$$

$$\boxed{f'(x) = -3(5x+1)^{-8/5}}$$

6. (8 pts) Find $f''(x)$ and $f''(-2)$ given $f(x)=\frac{-3}{4x^3}$. $= \frac{-3}{4} x^{-3}$

$f'(x) = \frac{-3}{4}(-3)x^{-4} = \frac{9}{4}x^{-4}$

$f''(x) = \frac{9}{4}(-4)x^{-5} = -9x^{-5} = \frac{-9}{x^5}$

$f''(-2) = \frac{-9}{(-2)^5} = \frac{-9}{-32} = \boxed{\frac{9}{32}} \approx 0.28125$

7. (16 pts total) Find the derivative using the appropriate rule.

A. $f(t)=4t^{-1/2}-\frac{9}{\sqrt[3]{t}}-\sqrt[5]{10t^2}$ $= 4t^{-1/2} - 9t^{-1/3} - (10t^2)^{1/5}$

$f'(t) = 4\left(\frac{-1}{2}\right)t^{-3/2} - 9\left(\frac{-1}{3}\right)t^{-4/3} - \frac{1}{5}(10t^2)^{-4/5}(20t)$

$\boxed{f'(t) = -2t^{-3/2} + 3t^{-4/3} - 4t(10t^2)^{-4/5}}$

B. $g(x)=(\overset{f}{2x+1})^3(\overset{g}{2x-1})^4$ $f'g+g'f$

$g'(x) = 3(2x+1)^2(2)(2x-1)^4 + 4(2x-1)^3(2)(2x+1)^3$

$\boxed{g'(x) = 6(2x+1)^2(2x-1)^4 + 8(2x-1)^3(2x+1)^3}$

***BONUS** (total of 15 extra points)*

A. (5 pts) Write the equation for the tangent line to the curve $f(x) = 2x^2 + 3x + 1$ at $x = -2$. Write the equation in slope-intercept form.

$m = f'(-2) = 4x + 3$

$y - y_1 = m(x - x_1) \quad (-2, 3)$

$f'(-2) = -8 + 3 = -5$

$y - 3 = -5(x - (-2))$

$y - 3 = -5x - 10$

$+3 \qquad +3$

$\boxed{y = -5x - 7}$

B. (10 pts) A rocket can rise to a height $h(t) = t^3 + 0.5t^2$ feet in t seconds.

i. Find its velocity 10 seconds after it is launched.

$v(t) = h'(t) = 3t^2 + 0.5(2t) = 3t^2 + t$

$v(10) = 3(10)^2 + 10 = \boxed{310 \frac{FT}{SEC}}$

ii. Find its acceleration 10 seconds after it is launched.

$a(t) = v'(t) = 6t + 1$

$a(10) = 6(10) + 1 = \boxed{61 \frac{FT}{SEC^2}}$